Solution:

Because $\mathbb{R}^3$ is isomorphic to $\mathbb{P}_2$ we have a unit vector basis for $\mathbb{P}_2$ (i.e. three linearly independent vectors that span, all with Length $=1$) written as $\{1, x, x^2\}$

The equivalent in $\mathbb{R}^3$ is $\{(1,0,0), (0,1,0), (0,0,1)\}$

Lets rewrite P_1, P_2, P_3 as vectors in $\mathbb{R}^3$ like so

$$P_1 = -x + 1$$

$$= 1 + -x + 0x^2$$

$$= \begin{bmatrix}1\\0\\0\end{bmatrix}1 + \begin{bmatrix}0\\-1\\0\end{bmatrix}x + \begin{bmatrix}0\\0\\0\end{bmatrix}x^2$$

$$= \begin{bmatrix}1\\-1\\0\end{bmatrix}, \text{ similarly, } P_2 = x + 2 = \begin{bmatrix}2\\1\\0\end{bmatrix}, \quad P_3 = x^2 + 1 = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

Together, these three vectors in matrix form...

Reduce like so, $\begin{bmatrix}1 & 2 & 1\\-1 & 1 & 0\\0 & 0 & 1\end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix}0 & 3 & 1\\-1 & 1 & 0\\0 & 0 & 1\end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix}-1 & 1 & 0\\0 & 3 & 1\\0 & 0 & 1\end{bmatrix}$

Shows three "pivots", which are instances of linear independence by way of basis formation, i.e. dimensional expansion, one per pivot, Thus, $\{P_1, P_2, P_3\}$ spans $\mathbb{P}_2$ with three linearly independent vectors that span $\mathbb{R}^3$, As $\mathbb{P}_2$ is isomorphic to $\mathbb{R}_3$, $\{P_1, P_2, P_3\}$ forms a basis for $\mathbb{P}_2$.

Now we can solve $x^2 + 2x = \alpha P_1 + \beta P_2 + \gamma P_3$ in a similar fashion.

$$x^2 + 2x = \alpha\begin{bmatrix}1\\-1\\0\end{bmatrix} + \beta\begin{bmatrix}2\\1\\0\end{bmatrix} + \gamma\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$x^2 + 2x = \begin{bmatrix}\alpha\\-\alpha\\0\end{bmatrix} + \begin{bmatrix}2\beta\\\beta\\0\end{bmatrix} + \begin{bmatrix}\gamma\\0\\\gamma\end{bmatrix}$$

$$x^2 + 2x = \begin{bmatrix}\alpha + 2\beta + \gamma\\-\alpha + \beta\\\gamma\end{bmatrix}$$

Finished on reverse →