$$x^2+2x = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha + 2\beta + \gamma \\ -\alpha + \beta \\ \gamma \end{bmatrix}$$

We know $\boxed{\gamma = 1}$ By looking at the third row, Now Let's look at the first row...

So.. $0 = \alpha + 2\beta + 1$

$* \; \alpha = -2\beta - 1$

* Substitute in for row 2

$$2 = -\alpha + \beta$$

$$2 = -(-2\beta - 1) + \beta$$

$$2 = 2\beta + 1 + \beta$$

$$1 = 3\beta$$

$\boxed{\beta = 1/3}$, so $2 = -\alpha + 1/3$ $\nearrow$ $5/3 = -\alpha$

$6/3 = -\alpha + 1/3$ $\quad \boxed{\alpha = -5/3}$

Check:

$$x^2 + 2x = -\frac{5}{3}(P_1) + \frac{1}{3}(P_2) + 1(P_3)$$

$$x^2 + 2x = -\frac{5}{3}(-x+1) + \frac{1}{3}(x+2) + 1(x^2+1)$$

$$= \frac{5}{3}x - \frac{5}{3} + \frac{1}{3}x + \frac{2}{3} + x^2 + 1$$

$$= \frac{5}{3}x + \frac{1}{3}x + x^2 - \frac{5}{3} + \frac{2}{3} + 1$$

$$= \frac{6}{3}x + x^2 - \frac{5}{3} + \frac{2}{3} + 1$$

$$= 2x + x^2 - \frac{3}{3} + 1$$

$$= 2x + x^2$$

$$= x^2 + 2x$$ ✓✓

— Mike DV :)